PART IV: CONE CONTRAST AND OPPONENT MODULATION COLOR SPACES

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Introduction

Cone excitation diagrams were introduced in Chapter 7. The advantage of using such a color space is that cones represent the initial encoding of light by the visual system. Because cone excitation is proportional to the quantal absorption rates of the three types of cone photopigments, it easier to think about how subsequent visual mechanisms combine and process cone signals. Fortunately there is consensus on the current estimates of cone spectral sensitivities (See Chapter 7).

The same logic that justifies the use of a cone excitation space can be applied to develop color spaces that represent explicitly the responses of subsequent visual mechanisms. Two ideas about the nature of retinal processing have been widely used in this fashion. The first is that photopigment excitations are recoded as contrast signals, so that the information available for further processing is provided in relative rather than absolute form. The second (as seen in Figure 7.1) is that signals from individual classes of cones are combined into three post-receptoral channels: one summative and two color-opponent. A color space based on the first idea alone is referred to as a cone *contrast space.* Color spaces based on both ideas are referred to as *opponent modulation spaces.*

One widely used opponent modulation color space was introduced explicitly by Derrington, Krauskopf, and Lennie (1984) based in part on ideas suggested by MacLeod and Boynton (1979) and by Krauskopf, Williams and Heeley (1982). This implementation of an opponent modulation space is here referred to as the DKL color space. (In Chapter 7 it is called the DKL cone excitation space.) Many of the ideas key to understanding the DKL space can be understood more simply in the context of cone contrast space. For this reason, we begin with a discussion of cone contrast space and then build on this material to develop the DKL space. We assume that the reader is familiar with the mathematics of color vision as presented in Part III of this Appendix.

Cone Contrast Space

The use of cone contrast space (or opponent modulation space) makes sense primarily when there is a background with respect to which contrast may be computed. Two typical stimulus configurations where this prerequisite holds are shown schematically in Figure A.4.1, where the stimuli being described are modulations of a large uniform background. Panel (a) shows a spot increment/decrement; panel (b) shows a sinusoidal modulation. We use the term increment/decrement to describe the spot stimulus because it is possible for a spot to be an increment for one cone type and a decrement for another. As shown in the figure, we let the vector $(P_{LO}\, P_{MO}\, P_{SO})^T$ represent the cone excitation coordinates of the background and the vector $(P_L\, P_M\, P_S)^T$ represent the cone coordinates of the stimulus region we wish to describe.[1] We define the differential cone excitation coordinates $(\Delta P_L\, \Delta P_M\, \Delta P_S)^T$ as the vector (entry-by-entry) difference between $(P_{LO}\, P_{MO}\, P_{SO})^T$ and $(P_L\, P_M\, P_S)^T$.

Transforming to Cone Contrast Coordinates

The transformation between cone excitation coordinates and cone contrast coordinates is given by equation A.4.1.

$$(C_L\, C_M\, C_S)^T = \left(\frac{\Delta P_L}{P_{LO}}\ \frac{\Delta P_M}{P_{MO}}\ \frac{\Delta P_S}{P_{SO}}\right)^T \qquad \text{(A.4.1)}$$

where $(C_L\, C_M\, C_S)^T$ is simply a vector of the conventional contrasts seen by each class of cone. Panels (a) and (b) of Figure A.4.2 show how a stimulus of the sort shown in panel (a) of Figure A.4.1 can be represented graphically in cone excitation space and in cone contrast space. These sorts of geometric representations make natural the terminology in which colored stimuli are referred to as points in color space. The transformation from excitation to contrast space may be thought of as a shift in the origin followed by a rescaling of the axes.

The key feature of cone contrast space is that it incorporates a simple Von Kries/Weber normalization model of retinal processing into the stimulus representation. Von Kries (1905) suggested that signals were normalized independently in the three separate cone pathways. Weber's Law may be understood as stating that in each pathway the normalization takes the specific form of Equation A.4.1. This model has the effect of equating stimuli across different choices of background. An example is shown in panel (c) of Figure A.4.2. Because cone contrast coordinates depend on the background, a crucial component of using cone contrast coordinates is to specify the background. Without this additional information, it is not possible to determine the excitation coordinates of the stimulus from the contrast coordinates.

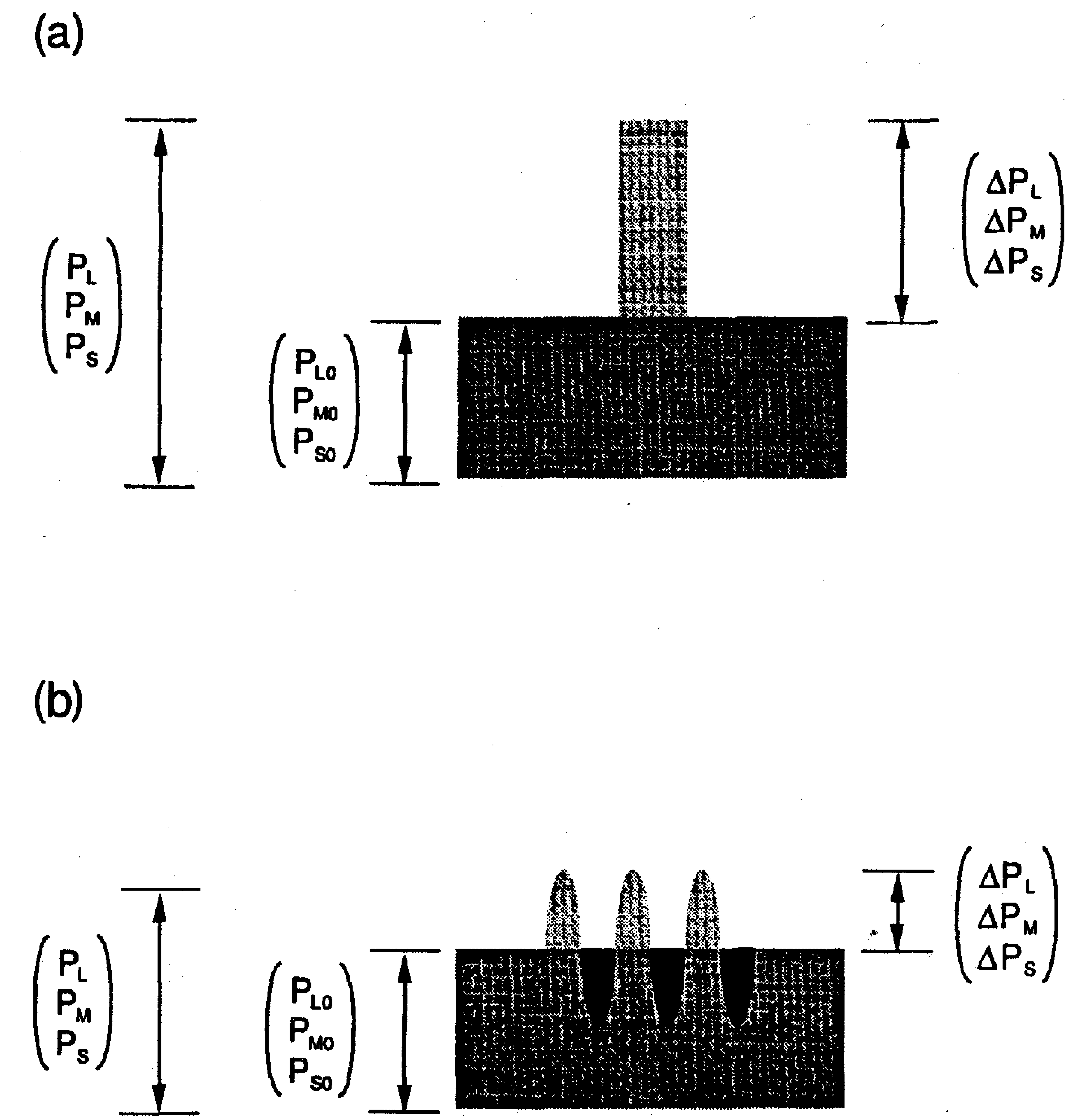

Figure A.4.1 Two stimulus configurations for which a cone contrast (or opponent modulation) stimulus description is appropriate. Panel (a) An increment/decrement is presented on a uniform background. The increment/decrement and the background do not necessarily have the same spectral composition, and the same stimulus may be an increment for one type of cone and a decrement for another. The cone excitation coordinates of the background are $(P_{L0}\,P_{M0}\,P_{S0})^T$. The cone excitation coordinates of increment/decrement plus the background are given by $(P_L\,P_M\,P_S)^T$. We define the differential cone excitation coordinates $(\Delta P_L\,\Delta P_M\,\Delta P_S)^T$ as the vector difference between $(P_{L0}\,P_{M0}\,P_{S0})^T$ and $(P_L\,P_M\,P_S)^T$. Panel (b) A uniform background is modulated sinusoidally. The cone excitation coordinates of the background are $(P_{L0}\,P_{M0}\,P_{S0})^T$, while the cone excitation coordinates at the peak of the modulation are $(P_L\,P_M\,P_S)^T$. The differential cone excitation coordinates $(\Delta P_L\,\Delta P_M\,\Delta P_S)^T$ are again the vector difference between $(P_{L0}\,P_{M0}\,P_{S0})^T$ and $(P_L\,P_M\,P_S)^T$. When the stimulus is a spatial or temporal modulation, it is conventional to use the differential cone excitation coordinates calculated for the modulation peak.

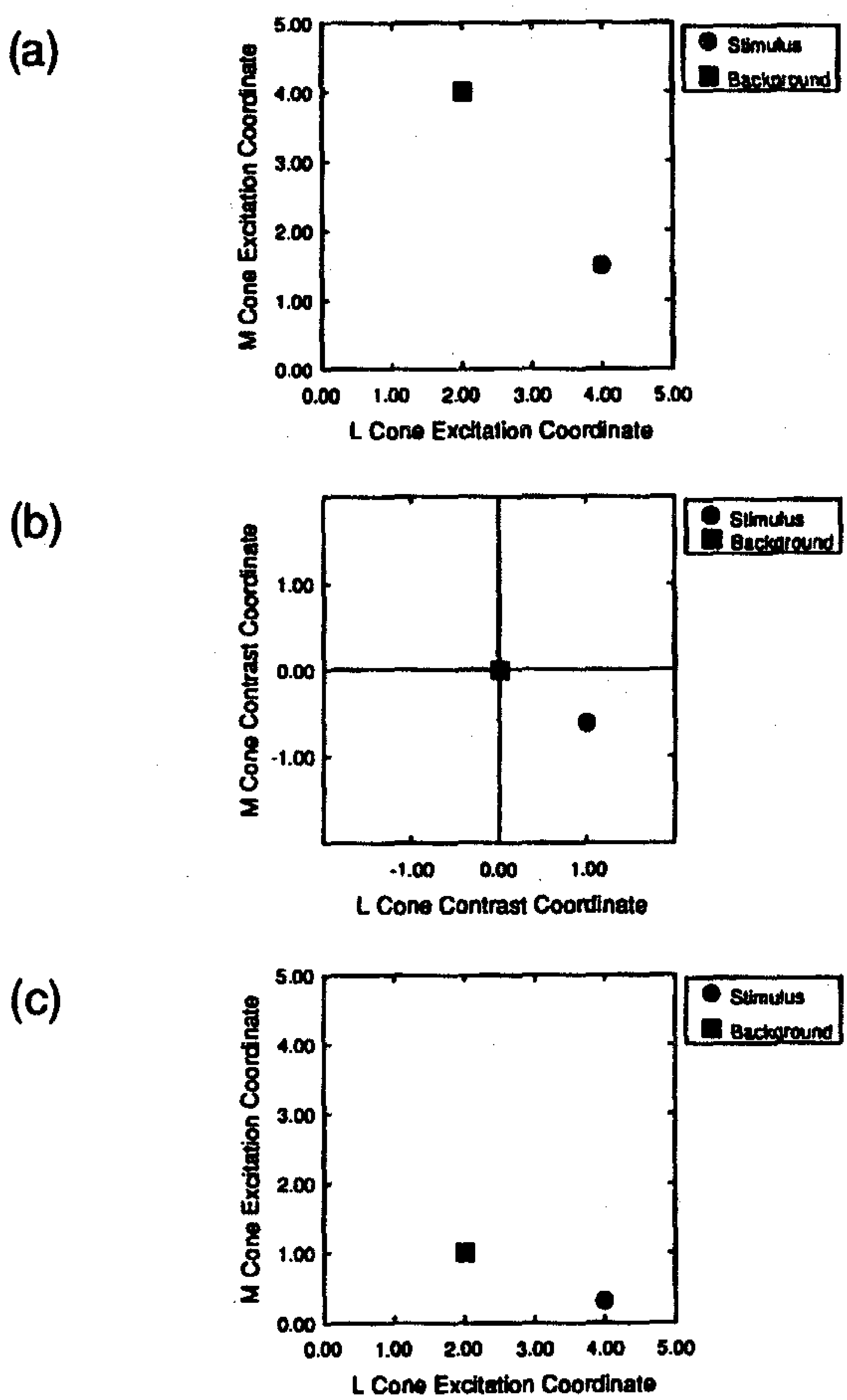

Figure A.4.2 Graphical representation of cone excitation and cone contrast coordinates for a spot increment/decrement. For graphical simplicity, only a two-dimensional plot of the L- and M-cone coordinates is shown. To visualize three-dimensional cone coordinates, one must either provide several two-dimensional views or use some other graphical technique which shows the full three-dimensional structure. Panel (a) Cone excitation coordinates. The closed square shows the L- and M-cone excitation coordinates of the uniform background (2.0,4.0). The closed circle shows the L- and M-cone excitation coordinates of the spot stimulus (4.0,1.5). Note that the spot is an L-cone increment and M-cone decrement. Panel (b) Cone contrast coordinates for the same stimulus. The background always plots at the origin in a cone contrast diagram, and it is often omitted from plots. The stimulus has positive L-cone contrast and negative M-cone contrast (1.0,–0.625). Panel (c) Cone excitation coordinates for a different spot stimulus (4.0,0.375) against a different background (2.0,1.0). The physical difference between this stimulus and the stimulus depicted in panel (a) is clear in the cone excitation space. The two stimuli have identical representations in cone contrast space.

Whether to represent stimuli in cone excitation or in cone contrast space depends in large measure on the extent to which the investigator wishes to accept the Von Kries/Weber normalization model as a starting point for further thinking. The representational decision may depend in large measure on which space best brings out the regularities of a particular data set.

The definition of cone contrast coordinates is straightforward when there is an unambiguous background. For complex stimuli (in particular for natural images) the definition is less clear. The difficulty arises in deciding what cone excitation coordinates $(P_{LO}\, P_{MO}\, P_{SO})^T$ to use for normalization. One possible choice is the spatial average of the cone excitation coordinates at each stimulus location (Buchsbaum, 1980; Brainard and Wandell, 1986; D'Zmura and Lennie, 1986; Land, 1986). Another is the cone excitation coordinates of the brightest location in the stimulus (Land and McCann, 1971). There is no guarantee, however, that either choice correctly models early visual processing. For this reason, investigators studying performance for complex stimuli have tended to use cone excitation coordinates.

A Metric for Contrast

Consider two stimulus modulations whose cone contrast coordinates differ only by a single scalar, that is modulations A and B such that $(C_L^A\, C_M^A\, C_S^A)^T = k(C_L^B\, C_M^B\, C_S^B)^T$ for some constant k. We say that two such modulations are in the same direction in color space and differ only in their signal strength. Cone contrast is a natural measure of signal strength for modulations that isolate a single class of cone. For stimuli that modulate multiple cone classes, it is not clear that there is a generalization of the concept of contrast that would allow us to summarize signal strength with a single number. There is a great temptation to define contrast for arbitrary modulations, however. For example, if we have measured the spatial contrast sensitivity functions (CSFs) for modulations in several different directions in cone contrast space, it would be convenient to compare the CSFs by plotting them all against a single contrast axis.[2] But how should this contrast axis be defined for modulations in different color directions? There is currently no agreed upon answer to this question.

One possible choice is to define contrast for any color direction to have unit value at the detection threshold for modulations in that direction. This principle has the attractive feature that it equates the visual responses based on direct measurements. But detection thresholds vary between observers, with the background, and with the spatial and temporal properties of the stimulus, so this method is not practical in general. A related possibility is to define contrast to have unit value at the detection threshold for an ideal observer (Geisler, 1989; Sekiguchi, Williams and Brainard, 1993). This eliminates observer variability from the definition, but requires instead standardization of parameters such as the relative number of L-, M-, and S-cones present in the retina. Both methods seem appropriate for particular studies but too unwieldy for general use. Neither method is likely to coincide with the natural definition of contrast for cone isolating stimuli.

One convenient convention for specifying the contrast of a modulation in an arbitrary color direction is to compute the pooled cone contrast as shown in Equation A.4.2 (Chaparro *et al.*, 1993).

$$C = \sqrt{C_L^2 + C_M^2 + C_S^2} \tag{A.4.2}$$

This quantity is the square-root of the cone contrast energy and is closely related to the vector-length model of color thresholds (Poirson, Wandell, Varner and Brainard, 1990). Using pooled cone contrast as a measure of signal strength has the attractive feature that it is independent of apparatus, observer, and stimulus configuration details. It has the slightly peculiar feature that the maximum physically achievable contrast for an isochromatic modulation (that is a modulation where the differential cone excitation coordinates have the same chromaticity as the background) is $C_{Max} = \sqrt{3}$ rather than the conventional $C_{Max} = 1$. This may be remedied by modifying Equation A.4.2 to define the pooled cone contrast as $C = \sqrt{(C_L^2 + C_M^2 + C_S^2)/3}$, but this leads to the similar oddity that the maximum physically achievable contrast for cone isolating stimuli is limited at $C_{Max} = 1/\sqrt{3}$.

The fact that it is difficult to define a single number measure of modulation stimulus strength across color directions serves to remind us that chromatic signal strength is unlikely to be univariate. At the very least, investigators should be cautious about experimental designs or conclusions that depend critically on how the contrast is scaled across color directions.

DKL Space

The DKL color space shares with cone contrast space the feature that it is based on a model of early visual processing (See p. 251 and Figure 7.1). The model starts with the assumption that early processing extracts differential cone signals. Once extracted, however, the differential cone signals are not simply rescaled but rather recoded by three post-receptoral mechanisms: a luminance mechanism and two opponent chromatic mechanisms. DKL coordinates represent the responses of these hypothesized mechanisms. To understand how to represent stimuli in DKL space, it is necessary to understand how the responses of these mechanisms are computed. Indeed, in this appendix, I define the DKL space by specifying the response properties of the underlying mechanisms. This development is atypical. Flitcroft (1989) provides a clear example of the more conventional development, which defines the space in terms of the stimuli that isolate the mechanisms. I emphasize the mechanism properties because I believe this approach makes explicit the model underlying the DKL space. The two developments are formally equivalent (see Knoblauch, 1995). As we will see below, we can derive the mechanism-isolating modulations once we specify the mechanism properties.

Luminance Mechanism

The luminance mechanism is defined so that its response is proportional to the photopic luminance of the differential stimulus. By inverting Equation A.3.14, we can derive the relation between differential cone excitation coordinates and differential tristimulus coordinates:[4]

$$\begin{pmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{pmatrix} = \begin{pmatrix} 2.9448 & -35001 & 13.1745 \\ 1.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 62.1891 \end{pmatrix} \begin{pmatrix} \Delta P_L \\ \Delta P_M \\ \Delta P_S \end{pmatrix}. \qquad \text{(A.4.3)}$$

The second row of this matrix equation tells us that the differential response of the luminance mechanism (denoted by ΔR_{Lum}) is given by

$$\Delta R_{Lum} = k_{Lum}\ (W_{Lum,L}\, \Delta P_L + W_{Lum,M}\, \Delta P_M + W_{Lum,S}\, \Delta P_S) \qquad \text{(A.4.4)}$$

where $W_{Lum,L} = 1.000$, $W_{Lum,M} = 1.0000$, and $W_{Lum,S} = 0.0000$. The notational choice "R" is a mnemonic for "response" while the notational choice "W" is a mnemonic for "weight" as in "weighted sum." The actual values for the weights come from the second row of the matrix in Equation A.4.3. The constant k_{Lum} defines the units for the mechanism response.[5]

L-M Opponent Mechanism

The first chromatic mechanism is referred to as the L-M opponent mechanism. Unlike the luminance mechanism, which is defined directly in terms of its weights, the L-M opponent mechanism is defined by two properties it must satisfy. First, it is a chromatic mechanism, so that its response is zero when the differential signal has the same chromaticity as the background. That is, the response of the mechanism is zero when

$$(\Delta P_L \; \Delta P_M \; \Delta P_S)^T = k \; (P_{LO} \, P_{MO} \; P_{SO})^T \qquad \text{(A.4.5)}$$

for any constant k. Second, the mechanism response is not affected by the excitation of the S-cones. The general form for the response of the L-M opponent mechanism is

$$\Delta R_{L\text{-}M} = k_{L\text{-}M} \; (W_{L\text{-}M,L} \Delta P_L + W_{L\text{-}M,M} \, \Delta P_M + W_{L\text{-}M,S} \, \Delta P_S \,). \qquad \text{(A.4.6)}$$

To specify the L-M opponent mechanism, we must find weights $W_{L\text{-}M,L}$, $W_{L\text{-}M,M}$, and $W_{L\text{-}M,S}$ so that the two defining properties are satisfied. From the second defining condition we have $W_{L\text{-}M,S} = 0$. To satisfy the first condition, we plug in the values P_{L0}, P_{M0}, and P_{S0} for ΔP_L, ΔP_M, and ΔP_S in Equation A.4.6 and set the result to 0. By using the fact that $W_{L\text{-}M,S} = 0$ we obtain $W_{L\text{-}M,L} \, P_{L0} + W_{L\text{-}M,M} \, P_{M0} = 0$ and derive that $W_{L\text{-}M,M} = (-W_{L\text{-}M,L} P_{L0}) / P_{MO}$. Note that the weights for the L-M opponent mechanism vary with the background. In this sense, the DKL space incorporates a very specific theory of adaptation. The constant defines the units for the mechanism response.

S-Lum Opponent Mechanism

The second chromatic mechanism is referred to as the S-Lum opponent mechanism. The S-Lum opponent mechanism is also defined by two properties it must satisfy. Like the L-M opponent mechanism, its response is zero when the differential signal has the same chromaticity as the background. The second property may be stated as follows. The response of the mechanism is zero when both the differential S-cone signal ΔP_S and the response of the luminance mechanism ΔR_{Lum} are zero. Together, these conditions give us that

$$\Delta R_{S\text{-}Lum} = k_{S\text{-}Lum} \; (W_{S\text{-}Lum,L} \Delta P_L + W_{S\text{-}Lum,M} \Delta P_M + W_{S\text{-}Lum,S} \Delta P_S) \qquad \text{(A.4.7)}$$

with $W_{S\text{-}Lum,L} = -W_{Lum,L}$, $W_{S\text{-}Lum,M} = -W_{Lum,M}$, and $W_{S\text{-}Lum,S} = -(W_{S\text{-}Lum,L}P_{L0} + W_{S\text{-}Lum,m}P_{M0})/P_{S0}$. As for the L-M opponent mechanism, the weights for the S-Lum opponent mechanism vary with the background. The constant $k_{S\text{-}Lum}$ defines the units for the mechanism response.

Conversion to DKL Space

The discussion above defines the weights for the three DKL mechanisms given any background. The weights let us calculate the mechanism responses $(\Delta R_{Lum}\ \Delta R_{L\text{-}M}\ \Delta R_{S\text{-}Lum})^T$ (up to the free unit constants k_{Lum}, $k_{L\text{-}M}$, and $k_{s\text{-}Lum}$) from the differential cone coordinates $(\Delta P_L\ \Delta P_M\ \Delta P_S)^T$. Matrix notation is used to express the calculation succinctly.[4] Each row of the matrix that relates $(\Delta P_L\ \Delta P_M\ \Delta P_S)^T$ to $(\Delta R_{Lum}\ \Delta R_{L\text{-}M}\ \Delta R_{S\text{-}Lum})^T$ should contain the weights for the corresponding mechanism. Collecting together the expressions for the weights derived above and bringing the free unit constants to the left-hand side, we obtain

$$\begin{pmatrix} \dfrac{\Delta R_{Lum}}{k_{Lum}} \\[2ex] \dfrac{\Delta R_{L\text{-}M}}{k_{L\text{-}M}} \\[2ex] \dfrac{\Delta R_S}{k_{S\text{-}Lum}} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\[2ex] 1 & \dfrac{-P_{L0}}{P_{M0}} & 0 \\[2ex] -1 & -1 & \dfrac{P_{L0}+P_{M0}}{P_{S0}} \end{pmatrix} \begin{pmatrix} \Delta P_L \\[2ex] \Delta P_M \\[2ex] \Delta P_S \end{pmatrix} \qquad \text{(A.4.8)}$$

Equation A.4.8 lets us compute DKL coordinates up to three free multiplicative constants for each mechanism. Note again that the construction of the conversion matrix depends on the cone excitation coordinates of the background.

Setting the Unit Constants

To compute DKL coordinates for any specific background, we must choose values for the constants k_{Lum}, $k_{L\text{-}M}$, and $k_{S\text{-}Lum}$. Setting these constants is closely related to the issue of how to define a color contrast metric. A natural choice for k_{Lum} is to set it so that ΔR_{Lum} expresses luminance contrast. There is no such natural choice for $k_{L\text{-}M}$ and $k_{S\text{-}Lum}$. In their original paper Derrington et al. (1984) choose these constants so that the two chromatic mechanism responses took on the value 1.0 at the maximum modulation obtainable within the gamut of their monitor. Although this is a natural choice for any particular monitor, it has the disadvantage that it makes the definition of the color space apparatus dependent. Other possible strategies include normalizing to real or ideal observer detection thresholds, as discussed for cone contrast space above. A final possibility is to normalize each mechanism to have unit response when it is excited in isolation by a stimulus with unit pooled cone contrast. This has the attractive feature that it is independent of apparatus, observer, and stimulus configuration details. It is the choice we adopt for the example below.

A Conversion Example

This section provides a worked example for computing the DKL coordinates of a stimulus. Table A.4.1 provides a listing of a MATLAB program that performs the example calculations in their entirety.

Suppose we wish to convert a stimulus with differential cone excitation coordinates $(\Delta P_L\ \Delta P_M\ \Delta P_S)^T = (2.0000\ \ -2.5000\ \ 1.0000)^T$ seen against a background with cone excitation coordinates $(P_{L0}\ P_{M0}\ P_{S0})^T = (2.0000\ \ 4.0000\ \ 3.0000)^T$ into DKL coordinates. (The numbers for this example were chosen arbitrarily. There is no guarantee that these differential coordinates can be achieved within the gamut of a physically realizable device.) Inserting the values for the background cone coordinates into Equation A.4.8 we have

$$\begin{pmatrix} \dfrac{\Delta R_{Lum}}{k_{Lum}} \\[2ex] \dfrac{\Delta R_{L-M}}{k_{L-M}} \\[2ex] \dfrac{\Delta R_S}{k_{S-Lum}} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 \\[2ex] 1 & -\dfrac{1}{2} & 0 \\[2ex] -1 & -1 & 2 \end{pmatrix} \begin{pmatrix} \Delta P_L \\[2ex] \Delta P_M \\[2ex] \Delta P_S \end{pmatrix}. \tag{A.4.9}$$

To set the normalization constants, we find the stimuli with unit pooled cone contrast that isolate each of the DKL mechanisms. The first step is to invert Equation A.4.9 to derive

$$\begin{pmatrix} \Delta P_L \\[2ex] \Delta P_M \\[2ex] \Delta P_S \end{pmatrix} = \begin{pmatrix} \dfrac{1}{3} & \dfrac{2}{3} & 0 \\[2ex] \dfrac{2}{3} & -\dfrac{2}{3} & 0 \\[2ex] \dfrac{1}{2} & 0 & \dfrac{1}{2} \end{pmatrix} \begin{pmatrix} \dfrac{\Delta R_{Lum}}{k_{Lum}} \\[2ex] \dfrac{\Delta R_{L-M}}{k_{L-M}} \\[2ex] \dfrac{\Delta R_S}{k_{S-Lum}} \end{pmatrix}. \tag{A.4.10}$$

The three columns of the matrix in Equation A.4.10 provide the differential cone coordinates of stimuli that isolate each of the DKL mechanisms. This is because in each column of the matrix are the differential cone coordinates obtained for substituting the three DKL vectors $(1.0000\ 0.0000\ 0.0000)^T$, $(0.0000\ 1.0000\ 0.0000)^T$, and $(0.0000\ 0.0000\ 1.0000)^T$ into the right-hand side of the equation. Thus the differential cone coordinates of stimuli that isolate the DKL mechanisms are $(0.3333\ 0.6667\ 0.5000)^T$, $(0.6667\ -0.6667\ 0.0000)^T$, and $(0.0000\ 0.0000\ 0.5000)^T$ for the luminance, L-M opponent, and S-Lum opponent mechanisms respectively. (This may be checked easily by plugging these three vectors into the right-hand side of Equation A.4.9 and verifying that each of the results has only one non-zero entry.) Derrington et al. (1984) referred to these stimuli as luminance, constant-B and constant R & G modulations. In Chapter 7 (Figure 7.20) they are referred to as the achromatic, constant-S, and constant L & M modulations. I prefer to call them isochromatic, red-green isoluminant, and S-cone isoluminant modulations. (The terminology and conventions for using the DKL color space are still evolving.)

The differential cone coordinates obtained from Equation A.4.10 do not have unit pooled cone contrast. In this example, we adopt the convention that the normalizing constants, k_{Lum}, $k_{L\text{-}M}$, $k_{S\text{-}Lum}$, be chosen so that mechanism-isolating stimuli with unit pooled cone contrast produce unit responses in the three DKL mechanisms. Normalizing each modulation obtained above, we get $(1.1547\ 2.3094\ 1.7321)^T$, $(1.7889\ -1.7889\ 0.0000)^T$, and $(0.0000\ 0.0000\ 3.0000)^T$ as the differential cone excitation coordinates of the stimuli that should generate unit response in each of the DKL mechanisms. (To compute pooled cone contrast, we divide differential cone excitation coordinates above by the cone excitation coordinates of the background and then apply Equation A.4.2.) We want to choose the scalars k_{Lum}, $k_{L\text{-}M}$, and $k_{S\text{-}Lum}$ so that when these three vectors are multiplied by the matrix in Equation A.4.9, the three corresponding mechanism responses ΔR_{Lum}, $\Delta R_{L\text{-}M}$, and ΔR_S are unity. The appropriate scalars are $k_{Lum} = 0.2887$, $k_{L\text{-}M} = 0.3727$, and $k_{S\text{-}Lum} = 0.1667$. Substituting the constants into Equation A.4.9 and simplifying gives us

$$\begin{pmatrix} \Delta R_{Lum} \\ \Delta R_{L\text{-}M} \\ \Delta R_S \end{pmatrix} = \begin{pmatrix} 0.2887 & 0.2887 & 0.0000 \\ 0.3727 & -0.1863 & 0.0000 \\ -0.1667 & -0.1667 & 0.3333 \end{pmatrix} \begin{pmatrix} \Delta P_L \\ \Delta P_M \\ \Delta P_S \end{pmatrix}. \qquad \text{(A.4.11)}$$

Performing this matrix multiplication for the vector $(\Delta P_L\ \Delta P_M\ \Delta P_S)^T = (2.0000\ \ -2.5000\ \ 1.0000)^T$ we obtain its DKL coordinates as $(\Delta R_{Lum}\ \Delta R_{L\text{-}M}\ \Delta R_S)^T = (-0.1443\ \ 1.2112\ \ 0.4167)^T$. To convert from DKL coordinates back to differential cone coordinates, we would use the inverse of Equation A.4.11:

$$\begin{pmatrix} \Delta P_L \\ \Delta P_M \\ \Delta P_S \end{pmatrix} = \begin{pmatrix} 1.1547 & 1.7889 & 0.0000 \\ 2.3094 & -1.7889 & 0.0000 \\ 1.7321 & 0.0000 & 3.0000 \end{pmatrix} \begin{pmatrix} \Delta R_{Lum} \\ \Delta R_{L\text{-}M} \\ \Delta R_S \end{pmatrix}. \qquad (A.4.12)$$

Graphical Representation and Spherical Coordinates

The DKL coordinates obtained above may be used to plot the stimulus modulation in the color space diagram shown in panel (A) of Figure 7.20. The first coordinate, –0.1443, would locate the stimulus below the isoluminant plane towards the –90° pole of the axis labeled achromatic; the second coordinate, 1.2112, would locate the stimulus towards the 0° pole of the axis labeled constant S-cone; the third coordinate, 0.4167, would locate the stimulus towards the 270° pole of the axis labeled constant L & M-cone. (The convention in Figure 7.20 is that the 270° pole represents the direction of increasing S-cone response.)

Modulations represented in this color space diagram are sometimes expressed in spherical coordinates. The angular azimuth and elevation are readily computed from the rectangular coordinates. With the sign conventions of Figure 7.20, we obtain $\phi = \arctan(-0.4167/1.2112) = -18.98°$ and $\theta = \arctan\left(-0.1443/\sqrt{(-0.4167)^2 + (1.2112)^2}\right) = -6.43$. It is important to note that these angular specifications depend on the normalization method used to define unit responses for the three DKL mechanisms. For this reason, angular specifications must be interpreted with great care.

Discussion

The DKL space is not simple to understand or to use. As with cone contrast space, its usefulness depends chiefly on whether it brings out regularities in experimental data. Indeed, most of the discussion of cone contrast space above applies to the DKL space as well. At a broad level, the model underlying the space clearly captures the opponent nature of color coding (Hurvich and Jameson, 1957). Understanding the exact nature of the opponent mechanisms (and whether there are only three) is a subject of much current interest (see for example Krauskopf, Williams and Heeley, 1982; Krauskopf, Williams, Mandler and Brown, 1986; Guth, 1991; Krauskopf and Gegenfurtner, 1992; Cole, Hine and McIlhagga, 1993; DeValois and DeValois, 1993; Poirson and Wandell, 1993; Chichilnisky, 1994; Webster and Mollon, 1995). The derivation of DKL space presented here may be generalized to define color spaces based on the responses of any three linear color mechanisms.

As with cone contrast space, proper interpretation of DKL coordinates requires an explicit specification of the cone excitation coordinates of the background. In addition, since there is no agreed upon standard for the normalization constants k_{Lum}, k_{L-M}, and k_{S-Lum}, these must be explicitly specified whenever the space is used.

The specification of the DKL mechanism weights used here (Equation A.4.3) depends on the relative scalings chosen for the L, M, and S-cone sensitivities. In particular, the scalings used in Equation A.3.14 are chosen so that photopic luminance of a stimulus is given by the sum of its L- and M-cone coordinates. Sometimes cone sensitivities are scaled so that the maximum sensitivity of each cone is equal to unity. Such scaling changes must be taken into account when deriving DKL coordinates. The space as conventionally defined also incorporates the simplifying assumption that S-cones do not contribute to photopic luminance. The Smith–Pokorny (Table A.3.4) estimates of the cone sensitivities are designed so that this assumption holds.

The Relation Between Mechanisms and Modulations

As mentioned above, the development here is atypical in that it defines the DKL space in terms of visual mechanism properties rather than in terms of the modulations that isolate the mechanisms. Advanced students may garner insight about the relation between the two approaches from Figure A.4.3, which shows graphically the relation between color mechanisms and the modulations which isolate them.

Notes

[1] The superscript "T" after a vector denotes vector transpose. It indicates that the vector should be treated as a column vector in any matrix operation. We use this notation for the inline expressions to conserve vertical space; we show column vectors explicitly in the matrix equations and figures where space permits.

[2] See Chapter 9 for more on spatial contrast sensitivity functions.

[3] To avoid the inconvenience of double subscripts in this appendix, we denote luminance with "Lum" rather than the CIE approved "L_v."

[4] See Endnote 6 in Appendix Part III for the operational rules of matrix algebra.

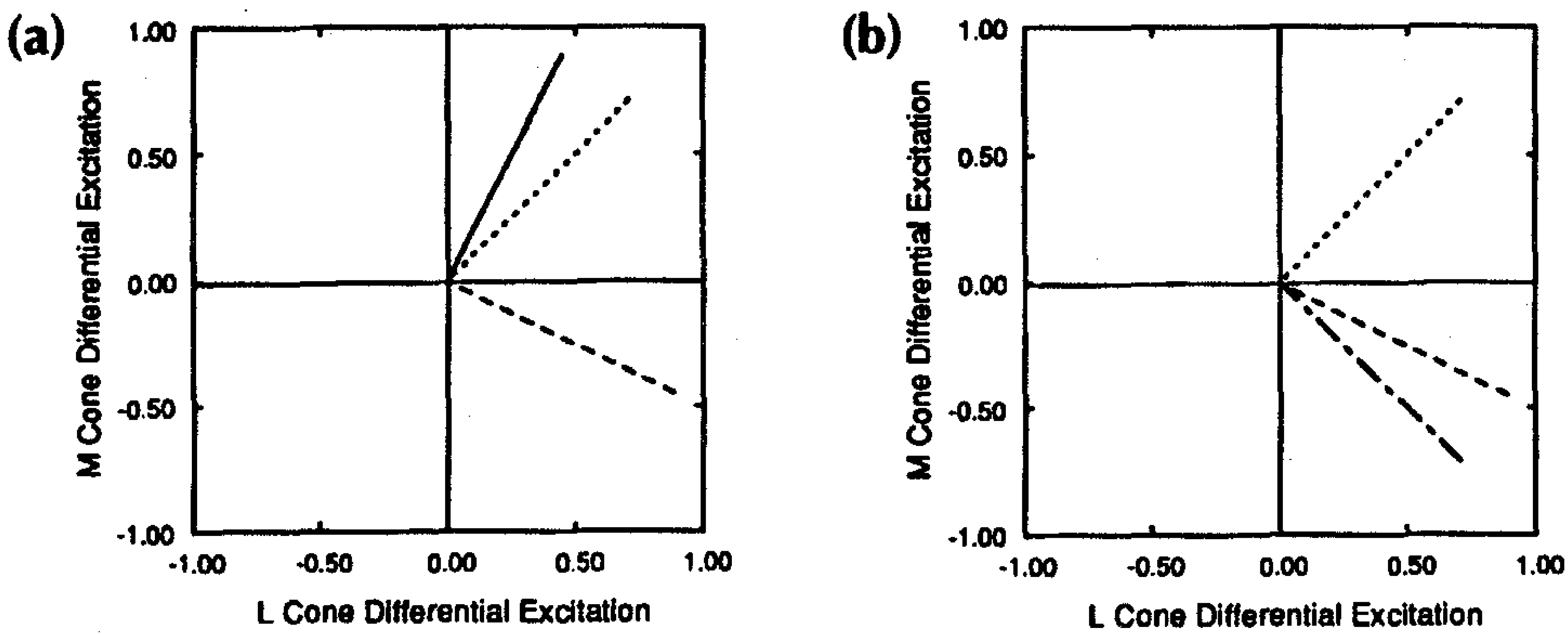

Figure A.4.3 Graphical representation of mechanisms and the modulations which isolate them. For graphical simplicity, only two-dimensional plots of the L- and M-cone coordinates are shown. For purposes of this figure, only color directions are of interest, therefore all vectors are shown normalized to unit length.

Panel (a) The color direction of an isochromatic modulation is shown by the solid line. Points on this line specify the differential L- and M-cone excitations that isolate the luminance mechanism. The direction of the line was obtained from the first column of the matrix in Equation A.4.12 and has the same relative cone coordinates as the background. The color direction of the L-M opponent mechanism is shown by the dashed line. Points on this line give the relative contribution of the differential L- and M-cone excitations to the mechanism response. The direction of this line was obtained from the second row of the matrix in Equation A.4.11. Note that the isochromatic stimulus is orthogonal to the L-M opponent mechanism. Readers familiar with analytic geometry will recognize that this orthogonality indicates that the mechanism response to the modulation is zero. If a full three dimensional plot were shown, the modulation would also be orthogonal to the direction of the S-Lum opponent mechanism. The color direction of the luminance mechanism is shown as the dotted line. The direction of this line was obtained from the first row of the matrix in Equation A.4.11. Note that the isochromatic stimulus does not line up with the luminance mechanism. The defining feature of the isochromatic modulation is its orthogonality to the chromatic mechanisms, not its relation to the luminance mechanism. The orthogonality is preserved under transformations of color space, whereas the angle between the modulation and the luminance mechanism is not.

Panel (b) The color direction of an isoluminant modulation that isolates the L-M opponent mechanism is shown by the dot-dash line. The direction of this line was obtained from the second column of the matrix in Equation A.4.12. As in panel (a) the color direction of the luminance mechanism is shown by the dotted line. Note that the isochromatic stimulus is orthogonal to the luminance mechanism. The color direction of the L-M opponent mechanism is again shown as the dashed line. Note that the isoluminant stimulus does not line up with the opponent mechanism it isolates.

Table A.4.1 The table provides a listing of a MATLAB program that computes the conversion example described in the appendix. MATLAB is a widely available numerical programming language. The interested reader may find this listing helpful in understanding the details of the color space conversion. Comments have been added to the listing in an attempt to make the program readable even for those unfamiliar with MATLAB syntax. MATLAB is a registered trademark of The MathWorks, Inc.

```
% DKL Example
%
% MATLAB program to compute the example
% used for the space in Appendix Part IV.
%
% MATLAB is a registered trademark of the
% MathWorks, Inc.
%
% 7/6/95 dhb Wrote it.

% STEP 1: Set the background vector for
% the conversion
bg = [2 4 3]';

% STEP 2: Set the vector we wish to convert
diffcone_coords = [2 -2.5 1]';

% STEP 3: Set M_raw as in equation A.4.9.
% This is found by inserting the background
% values into equation A.4.8. Different
% backgrounds produce different matrices.
% The MATLAB notation below just
% fills the desired 3-by-3 matrix.
M_raw = [ 1 1 0 ; ...
          1 -bg(1)/bg(2) 0 ; ...
         -1 -1 (bg(1)+bg(2))/bg(3) ];

% STEP 4: Compute the inverse of M for
% equation A.4.10. The MATLAB inv() function
% computes the matrix inverse of its argument.
M_raw_inv = inv(M_raw);

% STEP 5: Find the three isolating stimuli as
% the columns of M_inv_raw.  The MATLAB
% notation X(:,i) extracts the i-th column
% of the matrix X.
isochrom_raw = M_raw_inv(:,1);
rgisolum_raw = M_raw_inv(:,2);
sisolum_raw = M_raw_inv(:,3);

% STEP 6: Find the pooled cone contrast of each
% of these.  The MATLAB norm() function returns
% the vector length of its argument. The MATLAB
% ./ operation represents entry-by-entry division.
isochrom_raw_pooled = norm(isochrom_raw ./ bg);
rgisolum_raw_pooled = norm(rgisolum_raw ./ bg);
sisolum_raw_pooled = norm(sisolum_raw ./ bg);

% STEP 7: Scale each mechanism isolating
% modulation by its pooled contrast to obtain
% mechanism isolating modulations that have
```

(continued on next page)

Table A.4.1 *continued*

```
% unit length.
isochrom_unit = isochrom_raw / isochrom_raw_pooled;
rgisolum_unit = rgisolum_raw / rgisolum_raw_pooled;
sisolum_unit = sisolum_raw / sisolum_raw_pooled;

% STEP 8: Compute the values of the normalizing
% constants by plugging the unit isolating stimuli
% into A.4.9 and seeing what we get.  Each vector
% should have only one non-zero entry.  The size
% of the entry is the response of the unscaled
% mechanism to the stimulus that should give unit
% response.
lum_resp_raw = M_raw*isochrom_unit;
l_minus_m_resp_raw = M_raw*rgisolum_unit;
s_minus_lum_resp_raw = M_raw*sisolum_unit;

% STEP 9: We need to rescale the rows of M_raw
% so that we get unit response.  This means
% multiplying each row of M_raw by a constant.
% The easiest way to accomplish the multiplication
% is to form a diagonal matrix with the desired
% scalars on the diagonal. These scalars are just
% the multiplicative inverses of the non-zero
% entries of the vectors obtained in the previous
% step.  The resulting matrix M provides the
% entries of A.4.11.  The three _resp vectors
% computed should be the three unit vectors
% (and they are).
D_rescale = [1/lum_resp_raw(1) 0 0 ; ...
             0 1/l_minus_m_resp_raw(2) 0 ; ...
             0 0 1/s_minus_lum_resp_raw(3) ] ;
M = D_rescale*M_raw;
lum_resp = M*isochrom_unit;
l_minus_m_resp = M*rgisolum_unit;
s_minus_lum_resp = M*sisolum_unit;

% STEP 10: Compute the inverse of M to obtain
% the matrix in equation A.4.12.
M_inv = inv(M);

% STEP 11: Multiply the vector we wish to
% convert by M to obtain its DKL coordinates.
DKL_coords = M*diffcone_coords;

% STEP 12: convert to spherical coordinates.
% According to the conventions in the original DKL
% paper, azimuth of 0 is along our rgisolum axis,
% azimuth of 90 is along our negative sisolum
% axis.  The isochromatic axis has an elevation
% of 90 degrees.  To do the conversion, we flip the
% sign of the sisolum coordinate and then do a
% standard conversion to polar coordinates.
RADS_TO_DEGS = 360/(2*pi);
azimuth_rads = atan(-DKL_coords(3)/DKL_coords(2));
isolum_len = sqrt(DKL_coords(2)^2 + DKL_coords(3)^2);
elevation_rads = atan(DKL_coords(1)/isolum_len);
azimuth = RADS_TO_DEGS*azimuth_rads;
elevation = RADS_TO_DEGS*elevation_rads;
```

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